Revision of Beliefs with Perceived Experiences*

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Abstract

We consider a revision process of personal beliefs with perceived experiences. A player revises his old beliefs on his environment into new ones with the currently stored perceived experiences. A salient point of our theory is to take the personal experiences into belief revision. Both the beliefs and perceived experiences are formulated in the same mathematical manner. In this setting, we consider three kinds of belief changes, e-expansion, e-contraction and e-revision. We prove the Levi identity by these changes considering the perceived experiences, i.e., the e-revision is expressed in terms of the e-expansion and e-contraction. Thus, the revision of a player's personal view is characterized under his experiences.

JEL Classification: A12; C70; D83.

Keywords: Belief revision; Levi identity; Perceived experiences; Inductive game theory.

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1. Introduction

1.1. **Revision of beliefs: Methodology and Motivations.** This paper presents a revision process of a player's personal view on his social environment. In our theory, a player has initial beliefs on the environment. With the beliefs, he confronts the environment repetitively and stores information for some time as his *perceived experiences*. Finally he revises his initial beliefs according to the perceived experiences and derives his personal view on the environment. By the investigation, we describe an inductive derivation process of a player's personal view from his experiences.

Keneko and Kline [12, 13] (hereafter the KK theory) have recently been developing a new theory of *info-memory protocols*. Our theory employs Kaneko-Kline's framework as our basic context. The KK theory introduces players' memory functions and derives their various personal views according to the memory functions. We make use of the memory functions for the accumulation of information to derive their perceived experiences. While the KK theory shows a player's coherent personal view for his memory function, we show the revision process of the personal view based on the perceived experiences.

The situation we target is depicted in Figure 1, as formally stated in Section 3 and 4. We suppose that a player with initial beliefs $\mathbf{B} = \langle B, C_B \rangle$ encounters an environment repetitively. Here we call B his basic beliefs and C_B a causality set of the basic beliefs. Given his memory function, he obtains the perceived experiences $\mathbf{E} = \langle E, C(E) \rangle$, which consists of his basic experiences E and the causality set of the basic experiences $C(E)$ in the same mathematical manner as the beliefs. In the revision process, the player revises his old beliefs $\mathbf{B}$ in light of his perceived experiences $\mathbf{E}$ and obtains the new beliefs $\mathbf{B}'$.

For the revision of a player's beliefs, we consider three types of belief changes: *e-contraction*, *e-expansion*, and *e-revision*. By the *e-contraction*, a player eliminates a part of beliefs which he initially held but has not experienced. It means

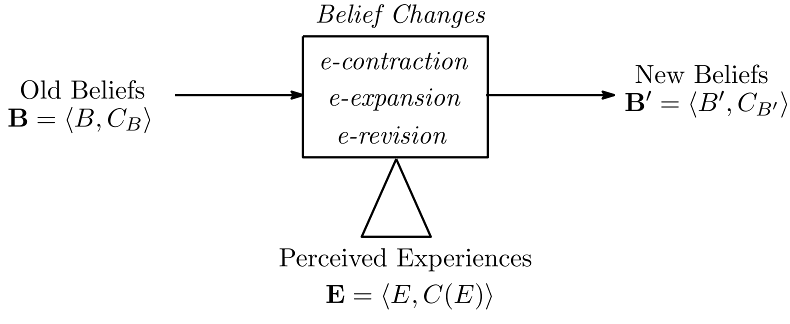

Figure 1. Revision Process

that his beliefs are not supported without his experiences. The *e-expansion* is to add some beliefs to reflect perceived experiences. These two belief changes are set-theoretically monotonous. However, the changes of our beliefs are not necessarily monotonous. We call the non-monotonous change according to the perceived experiences *e-revision.*

Our concept of e-revision is based on the Pareto principle: If an option O is no worse than an alternative O' in all dimensions of interest and better than O' in some, a player should prefer O. The revision concept based on the Pareto principle has been introduced by Schulte [18]. Our first result is to characterize the revision based on the Pareto principle by the e-contraction and e-expansion, which is called *Levi identity* in belief revision theory.[1] After showing this Levi identity as a benchmark of the e-revision, we will present some variations of the identity.

1.2. **Motivating example.** Plato [17] gave an *analogy of the cave* for explanation of the role of education. Our concept of the perceived experiences is similar to that of the *habituation* in the analogy. We explain the perceive experiences as compared with the habituation. And then we exemplify a brief description of our theory on the analogy.

The analogy of the cave can be summarized as follows: Prisoners have been confined in a cave from their birth, and they have been chained in such a way that all they can see is the back wall of the cave. On this wall, they can see shadows projected from the outside world. These shadows are the only real view of the world for them. Each prisoner constructs the own view of the world based on his memories and interpretations of the shadows.

We now consider the situation based on this analogy in Kaneko and Kline [12, 13]. Suppose that a prisoner observes the shadows every morning and every evening at a certain time. When the $m$orning comes, a Horse and a Cart always pass in turn, and the $e$vening comes as depicted in Figure 2.[2] To the prisoner who does not distinguish between the shadows, the world appears as in Figure 3, which shows four possible interpretations of two shadows passing by. Then how does he recognize the further situation? Plato continues the analogy as follows:

> Then there would be need of habituation, I take it, to enable him to see the things higher up. And at first he would most easily discern the shadows and, ... later, the things themselves, and from these he would go on to contemplate the appearance... (Plato [17, p. 748])

[1]Gärdenfors [4] provides the comprehensive account of belief revision theory.

[2]Each italic letter in the previous sentence means each symbol in the figures. E.g., '$m$orning represents m in the figures.

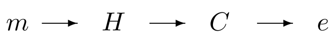

FIGURE 2. Objective World

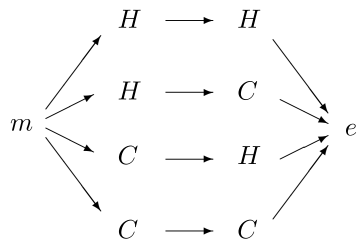

FIGURE 3. Personal View

The prisoner begins to notice the appearance of the shadows with the *habituation*. The perceived experiences are part of the habituation. Indeed, a player accumulates his information and observation as his perceived experiences. Such experiences will be got from a part of the habituation by further repetition.

Our theory expresses the situation that a player notices the appearance of the shadows as follows. Suppose that he notices that a horse first comes as his perceived experiences, i.e., he does not notice what comes second. Then his personal view would be changed as Figure 4. Namely, he would rule out the possibility that the first shadow of the day is a cart, and would reconstruct a new view that the first is a horse. In this paper, we propose a theory in which this process is broken down into several basic characteristic properties.

The reader may recall literature of *learning models* such as Fudenberg and Levine [3] and Kalai and Lehrer [8]. While those models permit a player to have a different view about his opponents' behaviors, our theory focuses on a construction of a personal view on a game itself. Moreover, Gilboa and Schmeidler [6] propose the other theory in which a player decides his behavior based on his past cases (the case-based decision theory). Unlike their case-based reasoning, our theory concentrates on the formation process of a personal view. The player of our theory decides his action after recognizing the environment he is facing now.

This paper is organized as follows: The next section introduces the info-memory protocols following the KK theory. In this section we also provide a player's experience constructed from his memories. Section 3 introduces a framework of revision in info-memory protocols. In Section 4, we present the fundamental revision structure with experience, *e-contraction*, *e-contraction*, and *the experience-based consistent revision* (e-revision). We also show the Levi identity in info-memory protocols. Section 5 investigates the construction of a personal view from the revision and discusses

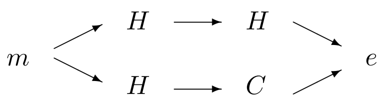

FIGURE 4. Revised Subjective View

the relation between the objective world and subjective views. Last section states the literatures of revision theory and gives comments on the further research.

2. Basic Context: An Info-memory Protocol

This section formalizes the info-memory protocols following Kaneko and Kline [12, 13]. The KK theory distinguishes between an objective world and subjective views. Both the objective world and subjective views are formulated in the info-memory protocols. In this section, we give a general framework of an info-memory protocol without any distinction between objective use and subjective use. After that, we define only the objective info-memory protocols. The subjective use of the protocols will be derived through the revision process in Section 5.

2.1. **Information protocols and Basic axioms.** Let W be a nonempty finite set of *information pieces*, A be a nonempty finite set of *actions*, and $\prec$ be a finite subset of $\bigcup_{m=0}^{\infty}((W \times A)^m \times W)$. Then an *information protocol* is given as a triple $\Pi = (W, A, \prec)$.

The relation $\prec$ is called a *causality relation*. When $m = 0$, we regard $(W \times A)^0 \times W$ as W. Then $\prec$ is a unary relation on W. Each element $\langle(w_1, a_1), \ldots, (w_m, a_m), w\rangle \in \prec$ is called a *feasible sequence*. We also write $[(w_1, a_1), \ldots, (w_m, a_m)] \prec w$ for its feasible sequence. $\langle \xi, w\rangle$ denotes a generic element of $\bigcup_{m=0}^{\infty}((W \times A)^m \times W)$, and $\langle w\rangle$ is that of $(W \times A)^0 \times W$, which is denoted by $\prec w$.

Based on a causal relation $\prec$, we give a partition on W. Let W^D be a set of *decision pieces*; $W^D := \{w \in W | \ [(w, a)] \prec v$ for some $a \in A$ and $v \in W\}$. $W^E := W \setminus W^D$ is called a set of *end pieces*.

Now let $N = \{0, 1, \ldots, n\}$ be the set of players to an information protocol Π. Player 0 is called the *chance* player and players $1, \ldots, n$ are called *personal* players. Then we provide a *player assignment* as a function $\pi : W \to 2^N$ that $\pi(w)$ assigns a single player for any $w \in W^D$ and $\{1, \ldots, n\}$ for any $w \in W^E$. Some single player is assigned for each decision piece and all the personal players are assigned for any end piece. We denote the set $\{w \in W | \ i \in \pi(w)\}$ as W_i, which is the set of player i's decision pieces and all the end pieces.

For a distinction between an objective description and a construction of a subjective view, Kaneko and Kline introduce the following four basic axioms and two non-basic axioms.

Axiom B1 (All Pieces used): For any $w \in W$, $\prec w$.

Axiom B2 (All Actions used): For any $a \in A$, there are some $u, v \in W$ such that $[(u, a)] \prec v$.

Give a sequence $[(w_1, a_1), \ldots, (w_k, a_k)]$, we define a *subsequence* by regarding each (w_t, a_t) as a component of the sequence to preserve the order. Also, we say that $\langle (w_1, a_1), \ldots, (w_k, a_k), w_{k+1} \rangle$ is a *subsequence* of $\langle (v_1, b_1), \ldots, (v_m, b_m), v_{m+1} \rangle$ if and only if either $[(w_1, a_1), \ldots, (w_k, a_k), (w_{k+1}, a)]$ is a subsequence of $[(v_1, b_1), \ldots, (v_m, b_m)]$ for some a or $[(w_1, a_1), \ldots, (w_k, a_k)]$ is a subsequence of $[(v_1, b_1), \ldots, (v_m, b_m)]$ and $w_{k+1} = v_{m+1}$. A *supersequence* is defined likewise.

Axiom B3 (Contraction): Let $\langle \xi, v \rangle$ be a feasible sequence and $\langle \xi', v' \rangle$ any subsequence of $\langle \xi, v \rangle$. Then $\langle \xi', v' \rangle$ is a feasible sequence.

Axiom B4 (Weak Extension): If $\xi \prec w$ and $w \in W^D$, then there are some $a \in A$ and $v \in W$ such that $[\xi, (w, a)] \prec v$.

Note that ξ' might be empty in Axiom B3. Then we regard $\langle v' \rangle$ as a feasible sequence if v' occurs in $\langle \xi, v \rangle$. Axiom B4 guarantees that a player can choose some action at each of his decision pieces.

When an information protocol satisfies the above four axioms, we call it a *basic information protocol*. Kaneko and Kline require two other non-basic axioms to be matched with classical extensive games. The basic axioms with non-basic axioms enable a complete description of an objective situation. We need to introduce more notations here to show these axioms:

- A feasible sequence $\langle \xi, v \rangle$ is *maximal* if there is no proper feasible supersequence of $\langle \xi, v \rangle$;
- $\langle (w_1, a_1), \ldots, (w_k, a_k), w_{k+1} \rangle$ for $k = 1, \ldots, m$ or $\langle w_1 \rangle$ is called an *initial fragment* of $\langle (w_1, a_1), \ldots, (w_m, a_m), w_{m+1} \rangle$;
- A *position* $\langle \xi, v \rangle$ is an initial fragment of some maximal feasible sequence. We denote the set of all positions by Ξ.

In order to represent an objective situation, we show two non-basic axioms.

Axiom N1 (Extension): If $\langle \xi, v \rangle$ is a position and $[(v, a)] \prec u$, then there is a $w \in W$ such that $\langle \xi, (v, a), w \rangle$ is a position.

Axiom N2 (Determination): Let $\langle \xi, v \rangle$ and $\langle \zeta, w \rangle$ be two positions. Then $\xi = \zeta$ implies $v = w$.

Note that N1 with B3 implies B4 (Lemma 2.4 in [12, p. 14]). Kaneko and Kline [12] show the equivalence between an extensive game in Kuhn [15] and an information protocol satisfying Axioms B1-B3, and N1-N2.

For an information protocol $\Pi = (W, A, \prec)$, we write $\Pi^o = (W^o, A^o, \prec^o)$ for an *objective information protocol*.

Example 2.1. We consider another variation of the 'analogy of the cave' in the Introduction. Now suppose that the shadows in a day depends on the weather depicted in Figure 5. In this objective world, two Horses pass in a $s$unny day and two Cart pass in a rainy day and then the $e$vening comes. Here, p represents the nature's action that time $p$asses.

Then the objective information protocol $\Pi^o = (W^o, A^o, \prec^o)$ is represented as follows: $W^o = \{m, H, C, e\}$ where $W^{oE} = \{e\}$ (i.e., e is the unique end piece in this protocol), $A^o = \{s, r, p\}$, and $\prec^o$ is all subthreads of the maximal sequences $\langle (m, s), (H, p), (H, p), e\rangle$ and $\langle (m, r), (C, p), (C, p), e\rangle$. Then this objective information protocol satisfies both the basic axioms and non-basic axioms.

Nevertheless, consider the personal view depicted in Figure 3 in this objective world by regarding each arrow as the nature's action p. For two positions $\langle (m, p), (C, p), C\rangle$ and $\langle (m, p), (C, p), H\rangle$ (or $\langle (m, p), (H, p), C\rangle$ and $\langle (m, p), (H, p), H\rangle$), Axiom N2 is violated. This is because $[(m, p), (C, p)]$ ($[(m, p), (H, p)]$, respectively) does not imply the same information piece. In this case, Axiom B2 is also violated because s, r are not used.

Furthermore look at Figures 6 and 7. While Figure 6 violates both Axioms B4 and N1, Figure 7 satisfies B4 but violates N1. Because, for a position $\langle (m, r), C\rangle$ and the feasible sequence $(C, p) \prec e$, $\langle (m, r), (C, p), e\rangle$ is not a position. Nevertheless $\langle (m, r), C\rangle$ forms another feasible sequence $\langle (m, r), (C, p), H\rangle$. That is, Axiom B4 gives successive feasible sequences and Axiom N1 guarantees that a root is uniquely determined.

2.2. **Memory functions and Accumulation of memories.** We introduce players' memory functions which gives a part of histories that every player knows at each of his decision pieces. Each of them accumulates his own memories and keeps them as his basic experiences. Consider an objective information protocol $\Pi^o = (W^o, A^o, \prec^o)$, which is not assumed to satisfy any particular axioms.

A memory of a player consists of recognized information pieces and actions in the past. We call the pair a *memory thread*. Let $A_v (\subseteq A)$ be a set of available actions at $v \in W$, then a memory thread is a finite feasible sequence

$$\mu = \langle (v_1, b_1), \ldots, (v_m, , b_m), v_{m+1}\rangle$$

where, for all $t = 1, \ldots, m$, $v_t \in W$, $b_t \in A_{v_t}$ and $v_{m+1} \in W$. Note that the KK theory gives a memory thread with feasible action set at each information piece. Our

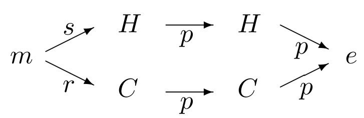

FIGURE 5. Objective World

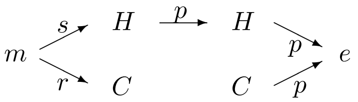

FIGURE 6. Violation of both B4 and N1

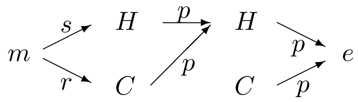

FIGURE 7. Violation of only N1

theory does not need the feasible action set and omits the set (cf. p. 10 in Kaneko and Kline [13]).

Each component (v_t, b_t) or v_{m+1} in μ is called a *memory knot* for the distinction of feasible sequences. The ending memory knot v_{m+1} of μ is denoted by $\epsilon(\mu)$ and is called the *tail* of μ.

Let Ξ_i be a subset of Ξ such that $\{\langle \xi, w\rangle \in \Xi \mid i \in \pi(w)\}$, i.e., player i's positions. Then a memory function is defined as follows:

Definition 2.2. A function m_i is called a *memory function* of player i if and only if, for each position $\langle \xi, w\rangle \in \Xi_i$, $\mathsf{m}_i\langle \xi, w\rangle$ is a finite non-empty set of memory threads satisfying $i \in \pi(\epsilon(\mu))$ for all $\mu \in \mathsf{m}_i\langle \xi, w\rangle$.

A memory function m_i stipulates that any information pieces at any tails in m_i belong to the set of i's information pieces. The memory function of player i takes i's all perceptions of the objective world, i.e., each player recognizes an objective world only through his memory function.

Now we give a basic framework for an objective world and players' recognition of it. An *objective info-memory protocol* is a triple $(\Pi^o, \pi^o, \mathsf{m}^o)$ with a set of players $N = \{1, \ldots, n\}$ satisfying Axioms B1-B3, and N1-N2, which consists of:

- an objective information protocol $\Pi^o = (W^o, A^o, \prec^o)$;
- an objective player assignment $\pi^o : W^o \to 2^N$;
- objective memory functions $\mathsf{m}^o = (\mathsf{m}^o_1, \ldots, \mathsf{m}^o_n)$.

The objective info-memory protocol provides a framework to describe the situation that the players repetitively play in a mutual situation. Each of them is supposed to accumulate memory threads through his memory function as defined below:

Definition 2.3. For $\Xi \subseteq \Xi^o_i$, $T(\mathsf{m}^o_i; \Xi)$ is i's accumulation set of his memory threads defined as follows:

$$T(\mathsf{m}^o_i; \Xi) = \bigcup_{\langle \xi, w\rangle \in \Xi} \mathsf{m}^o_i\langle \xi, w\rangle. \tag{1}$$

Since we define a memory thread without feasible action sets unlike the KK theory, the above accumulation does not have action sets. Therefore we do not need the operation which eliminates the action sets as the KK theory does.

In contrast to the KK theory, the union range Ξ in (1) is not necessarily equal to Ξ_i^o. A player does not perceive all the occurrence over his positions as his experience. That is, our theory is allowed the *partial accumulation* of his memory threads unlike the request of the perfect accumulation of the KK theory.[3] We do not discuss how the union range is decided. The decision may be related with a player's emotion, mentality, character, and so on. However, for the focus on the revision process, this paper concentrates on the accumulation of memory threads given Ξ

In order to compare a player's accumulations of memory threads with the feasible sequences in an information protocol, we need the following operation to obtain the subsequence-closed set. This operation helps the construction of a personal view discussed in Section 5.

Definition 2.4 (Basic Experience)**.** Given $\Xi \subseteq \Xi_i^o$, i's *basic experience* $E_i(\Xi)$ (or simply E_i) is the subsequence-closed set of memory threads as follows:

$$E_i(\Xi) = T^*(\mathsf{m}_i^o; \Xi), \tag{2}$$

where $T^*(\mathsf{m}_i^o; \Xi)$ be the set of all subsequences of every sequence in $T(\mathsf{m}_i^o; \Xi)$.

In the next section, we give the fundamental structure of a player's beliefs and experiences by using the above basic experience. Before proceeding the next section, we look at an example to understand the accumulations of memory threads and the operations.

Example 2.5. Recall the objective information protocol given in Example 2.1. For the objective information protocol with the set of player $N = \{i\}$, we give the objective player assignment as $\pi^o(w) = \{i\}$ for any $w \in W^o$.

Now let us suppose that he is aware of the difference of weather by the *habituation*: He finds two horses to come in a sunny day and two carts in a rainy day. In this paper, we do not discuss how the habituation is formed but consider how the habituation has an effect on personal views. Here we look at the effect of the habituation.

[3] See Kaneko and Kline [13, p. 11].

Consider the following memory function:

$$\mathsf{m}_i^o\langle \xi, w\rangle = \begin{cases} \{\langle m\rangle\} & \text{if } w = m \\ \{\langle (m,s), H\rangle\} & \text{if } \xi = (m,s) \\ \{\langle (m,r), C\rangle\} & \text{if } \xi = (m,r) \\ \{\langle (m,s),(H,p),H\rangle\} & \text{if } \xi = [(m,s),(H,p)] \\ \{\langle (m,r),(C,p),C\rangle\} & \text{if } \xi = [(m,r),(C,p)] \\ \left\{ \begin{array}{l} \langle (m,s),(H,p),(H,p),e\rangle, \\ \langle (m,r),(C,p),(C,p),e\rangle \end{array} \right\} & \text{if } w = e. \end{cases}$$

Player i recognizes the difference of weather under this memory function. If we suppose the perfect accumulation like the KK theory, the player completely accumulates all the above memory threads. In contrast, our theory investigates various habituations. For instance, if he focuses on a sunny day, i.e., $\Xi = \{\langle m\rangle, \langle (m,s), H\rangle, \langle (m,s),(H,p),H\rangle\}$, he accumulates

$$T(\mathsf{m}_i^o; \Xi) = \left\{ \begin{array}{l} \langle m\rangle, \langle (m,s), H\rangle, \\ \langle (m,s),(H,p),H\rangle \end{array} \right\}.$$

and

$$T^*(\mathsf{m}_i^o; \Xi) = \left\{ \begin{array}{l} \langle m\rangle, \langle H\rangle, \langle (m,s), H\rangle, \langle (H,p), H\rangle, \\ \langle (m,s),(H,p),H\rangle \end{array} \right\}.$$

This basic experience could break Axiom A4.

3. Event-causality Structure in the Info-memory Protocol

This section provides a fundamental structure of beliefs as the event-causality structure. A player's belief and experience are represented on the event-causality structure, as the belief structure and the perceive experience structure, respectively.

For the definition of the structure, we introduce *objective expressions* in the next subsection. By making use of this expression, we give semantics for the player's belief and experiences. That is, we put in a context each sequence of information protocol that the player has in mind.

3.1. **Objective expressions.** For a representation of players' beliefs, we consider a fixed set of *objective expressions*, denoted by $\boldsymbol{L}$. Objective expressions are combination of finite sequences in $\cup_{m=0}^{\infty}((W^o \times A^o)^m \times W^o)$ of a fixed objective information protocol $\Pi^o := (W^o, A^o, \prec^o)$. Formally, $\boldsymbol{L}$ consists of $\boldsymbol{A}$, $\boldsymbol{N}(\boldsymbol{A})$, and $\boldsymbol{P}(\boldsymbol{A})$ defined as follows:

(1) $\boldsymbol{A} := \cup_{m=0}^{\infty}((W^o \times A^o)^m \times W^o)$;
(2) $\boldsymbol{N}(\boldsymbol{A}) := \{\neg s \mid s \in \boldsymbol{A}\}$;
(3) $\boldsymbol{P}(\boldsymbol{A}) := \{s \rightsquigarrow t \mid s, t \in \boldsymbol{A}\}$.

The set $\boldsymbol{A}$ gives any finite sequence representable by the given objective information protocol. We call each member in $\boldsymbol{A}$ an *atomic objective expression.* $\boldsymbol{N}(\boldsymbol{A})$ gives a negation of each finite sequence in $\boldsymbol{A}$. We regard $\neg s$ for $s \in \boldsymbol{N}(\boldsymbol{A})$ as $s \in \boldsymbol{A}$. Therefore, while we can consider that the negation $\neg$ is infinitely used, the expression is restricted in two kinds, $s, \neg s$ for each $s \in \boldsymbol{A}$. We represent by $\neg s$ that s does *not* occur. For an atomic objective expression s, we refer s as the positive expression and $\neg s$ as the negative expression. Then we denote the positive expression of s in $\boldsymbol{A} \cup \boldsymbol{N}(\boldsymbol{A})$ as s^+.

The *possibility connective* $\rightsquigarrow$ represents a possibility that something may happen following a predecessor. That is, $s \rightsquigarrow t$ (s leads to t) means that, if s occurs, then t *may occur.* If a player has a belief $s \rightsquigarrow r$, then he believes that r may occur if s occurs.

In order to restrict the use of the possible connective to the relation between a supersequence and a subsequence, we set $\bar{\boldsymbol{P}}(\boldsymbol{A}) = \{s^+ \rightsquigarrow t^+ \in \boldsymbol{P}(\boldsymbol{A}) \mid \text{For } s, t \in \boldsymbol{A} \cup \boldsymbol{N}(\boldsymbol{A}),\ s^+ \text{ is a subsequence of } t^+\}$. Then $\bar{\boldsymbol{L}} \subset \boldsymbol{L}$ consists of $\boldsymbol{A}$, $\boldsymbol{N}(\boldsymbol{A})$, and $\bar{\boldsymbol{P}}(\boldsymbol{A})$.

The possibility connective may remind the reader of a modal concept in the modal logic or possible worlds in Kripke's semantics. Indeed, if a player has a belief, $s \rightsquigarrow r$, he believes the possibility r whenever he observes s. Nevertheless, he can change the possibility through the revision. That is, he refines several possibilities that he hold through the revision with his experiences. Unlike the modal logic, this paper looks at the revision process to refine the possibilities that a player holds.

We define semantics of each objective expression logically derived from other objective expressions. Consider the two-valued set $\{\top, \bot\}$ of beliefs. Then a player's belief assignment σ of the set $\boldsymbol{A}$ is a function $\sigma : \boldsymbol{A} \to \{\top, \bot\}$. We extend the assignment σ of a player to the function $\bar{\sigma} : \bar{\boldsymbol{L}} \to \{\top, \bot\}$ by the following types on the length of an objective expression:

- For any $s \in \boldsymbol{A}$, $\bar{\sigma}(s) = \top$ if and only if $\sigma(s) = \top$;
- $\bar{\sigma}(\neg s) = \top$ if and only if $\bar{\sigma}(s) = \bot$;
- $\bar{\sigma}(s \rightsquigarrow t) = \top$ if and only if $\bar{\sigma}(s) = \bot$ or $\bar{\sigma}(t) = \top$.

We interpret the above assignments as that he believes the expression to happen if a player assigns $\top$ to an objective expression and as that he does not believe it if he assigned $\bot$.

3.2. **Beliefs and Perceived Experiences.** The sequences in an info-memory protocol have been connected by the objective expressions above. As the next step, we give a player's beliefs and experiences in the objective expressions. For a finite subset

$F_1, F_2 \subseteq \boldsymbol{A} \cup \boldsymbol{N}(\boldsymbol{A})$, let $C(F_1; F_2)$ be the causality set between F_1 and F_2 defined as $C(F_1; F_2) = \left\{ s^+ \rightsquigarrow t^+ \in \bar{\boldsymbol{P}}(\boldsymbol{A}) \mid s \in F_1,\ t \in F_2 \right\}$. Then we define the fundamental structure, *the event-causality structure*, with the causality set as follows:

Definition 3.1 (Event-Causality Structure)**.** For a finite subset $F_1, F_2 \subseteq \boldsymbol{A} \cup \boldsymbol{N}(\boldsymbol{A})$, the *event-causality structure* is defined by a pair of $\langle F_1, C(F_1; F_2) \rangle$, where F_1 is called a *basic event*, F_2 is *a conceivable event*, and $C(F_1; F_2)$ is called the *causality set* between F_1, and F_2.

The event-causality structure provides the propositions based on the sequences and the relations among those propositions. Indeed, given $F_1, F_2 \subseteq \boldsymbol{A} \cup \boldsymbol{N}(\boldsymbol{A})$, $C(F_1; F_2)$ means the connections between two sequences as a causal relation $\prec$ in an information protocol does.

We introduce a player's beliefs and perceived experiences by the event-causality structure as follows:

- **Beliefs:** The event causality structure $\mathbf{B} = \langle B, C_B \rangle$ is called a player's *beliefs* if the positive expressions in B are subsequence-closed and there is a set B' with $B' \supseteq B$ such that $C_B := C(B; B')$. Then we call B the basic beliefs, and C_B a causality of B.
- **Perceived Experiences:** The event causality structure $\mathbf{E} = \langle E, C(E) \rangle$ is called *the perceived experiences* if $E = E(\Xi)$ as Definition 2.4[4] and $C(E) := C(E; E)$. Then we call E a player's basic experiences and $C(E)$ the causality of E.

While both the beliefs and the perceived experiences are defined on the event-causality structure, there is a difference between these two structures. First of all, while the conceivable event in the perceived experiences is the same E as the basic experiences, that of the beliefs is given as a set with $B' \supseteq B$. This means that he can recognize the relations in B also have some misunderstandings due to $B' \supseteq B$. A player also refines such misunderstandings through the revision.

Second, while the basic belief in the beliefs is represented by both the atomic objective expressions and the negation of them, the basic experience in the perceived experiences does not have any negation. This is because of the construction by a player's memory function (Recall Definition 2.4). However it does not mean that the player has no new negation through the revision. The acquisition of new negation in his belief is involved in the causality set and the consequences as defined below. In the following two sections, we investigate such revision process and the properties of it.

[4]A player's index i is abbreviated

In the repetitive process as Figure 1, a player inductively considers the basic events as the basic beliefs and basic experiences and derives the causality from the basic beliefs and experiences. Then the beliefs are revised with referring the perceived experiences and construct his personal view on his environment.

4. Revision of Beliefs with Perceived Experiences

So far we have provided some basic notions for the revision of a player's beliefs. By introduction of the objective expressions, we can represent the sequences in the info-memory protocols and give the event-causality structure for the beliefs and experiences. This section shows the belief change from $\mathbf{B}$ to the new belief $\mathbf{B}'$ according to $\mathbf{E}$ as in Figure 1. We introduce the contraction and expansion with the perceived experiences, called *e-contraction* and *e-expansion*, respectively. After that, we presents the fundamental axioms for the revision with the perceived experiences as *the experience-based consistent revision* (or *e-revision*) and characterize the revision by the e-contraction and e-expansion.

4.1. **E-contraction and E-expansion.** In the standard belief revision theory, there are three types of belief changes:

- A *contraction* is to eliminate some propositions without adding any proposition;
- An *expansion* is to add new propositions to the old belief without retracting any element of the old belief;
- A *revision* is to eliminate some propositions and to add new propositions in order to avoid contradictions in the new beliefs.

A contraction can be made when a player observes the proposition that contradicts what he previously believed. An expansion is commonly the result of new observations that a player has never watched. A revision is a change to avoid some contradictions by new observation but is not such monotonous changes as the contraction and expansion.

A common property of both a contraction and an expansion is set-theoretically monotonous. This is because each change rejects a new proposition if it is inconsistent with the propositions held initially. Nevertheless a player sometimes faces the situation where he gives up some proposition in the old belief and accepts another new proposition in the new belief. For instance, it is the occasion that he observes something unexpected or surprising. The revision can be made when he accepts the propositions inconsistent with the old beliefs without any inconsistency.

We apply these three changes to the revision with the perceived experiences. As defined before, the perceived experiences are not just an observation but the accumulation of the repetitive observations based on his memory function. A player changes his belief in the light of the accumulation.

We introduce the consequence set Cn:

Definition 4.1 (Consequence of Beliefs)**.** Let $\mathbf{B} = \langle B, C_B \rangle$ be a player's beliefs. *The consequence set* Cn of $\mathbf{B}$ is defined as follows:

$$\mathrm{Cn}(\mathbf{B}) = \left\{ v \in \bar{\boldsymbol{L}} \;\middle|\; \begin{array}{l} \bar{\sigma}(v) = \top \text{ for any } \bar{\sigma} \text{ with } \bar{\sigma}(u) = \top \\ \text{for all } u \in B \cup C_B. \end{array} \right\}.$$

This consequence set is the set of the believable propositions that a player induces from his belief structure. For instance, if the player believes $\{p, p \rightsquigarrow q\}$, then the consequence set guarantees that he induces $\{q\}$. Therefore he has two kinds of reasoning (i) the causality of his basic event set by the causality set and (ii) the consequence of his belief structure by the consequence set.

Now we define two set-theoretical operations for the event-causality structures: For the two event-causality structures $\mathbf{F}_1 = \langle F_1, C(F_1; F_1') \rangle$, $\mathbf{F}_2 = \langle F_2, C(F_2; F_2') \rangle$, $\mathbf{F}_1 \subseteq \mathbf{F}_2$ is $F_1 \subseteq F_2$ and $C(F_1; F_1') \subseteq C(F_2; F_2')$. In addition, let $\mathbf{E}^c$ be the structure $\langle B \setminus E, C(B \cap E; B \setminus E) \rangle$ for a player's beliefs $\mathbf{B} = \langle B, C_B \rangle$ and the perceived experiences $\mathbf{E} = \langle E, C(E) \rangle$. We represent by $\mathbf{E}^c$ what a player initially believed but did not experience. Indeed, the basic event $B \setminus E$ is the set of sequences that he initially held but did not have. And the causality $C(B \cap E; B \setminus E)$ is the objective expressions using the possibility connective that he did not have the causality even though he experienced the sequences he initially held. Since we take the difference between $\mathbf{B}$ and $\mathbf{E}$, the causality is a different from that of the event causality structure.

Now we define an *e-contraction* with the consequence set of beliefs:

Definition 4.2 (E-contraction)**.** Let $\mathbf{B} = \langle B, C_B \rangle$ and $\mathbf{B}' = \langle B', C_{B'} \rangle$ be two beliefs of a player, and $\mathbf{E} = \langle E, C(E) \rangle$ his perceived experiences. Then $\mathbf{B}'$ is *an e-contraction* from $\mathbf{B}$ with $\mathbf{E}$ if and only if

(C1) $\mathbf{B}' \subseteq \mathbf{B}$;

(C2) $\mathcal{E}^c \cap \mathrm{Cn}(\mathbf{B}') = \emptyset$;

(C3) there is no other belief structure $\mathbf{B}'' = \langle B'', C_{B''} \rangle$ such that $\mathcal{E}^c \cap \mathrm{Cn}(\mathbf{B}'') = \emptyset$ and $(B \cup C_B) \setminus (B'' \cup C_{B''}) \subsetneq (B \cup C_B) \setminus (B' \cup C_{B'})$,

where $\mathcal{E}^c = (B \setminus E) \cup C(B \cap E; B \setminus E)$.

This definition is a modified version of Schulte's *retraction-minimal contraction* (See Schulte [18, p. 346]). In his paper, Schulte proposes the revision principle based on the *Pareto principle* stated in Introduction. Indeed (C3) is based on this requirement, i.e., the change of beliefs should be minimized.

While (C1) is a natural condition to be a contraction, (C2) reflects our idea of the e-contraction. By definition, $\mathbf{E}^c$ is the set of the propositions that he initially held but did not experience. Therefore (C2) requires that, if a player does not have any experiences of the beliefs initially held, then he neither believes nor reasons them. We refer each member of $\mathbf{E}^c$ as a *non-experienced* proposition.

In general, a contraction in not decided uniquely. This is because there are usually several alternatives to contract when facing experiences inconsistent with his beliefs. For example, suppose that he believes $\{\neg q, p \rightsquigarrow q\}$ and his experience $\{p\}$. Then he might change his belief to either $\{p, \neg q\}$, $\{p, p \rightsquigarrow q\}$, or $\{p\}$. Nevertheless, it is uniquely determined in our theory because of the separation of the basic event and the causality.

Proposition 4.3. *Let* $\mathbf{B} = \langle B, C_B\rangle$, $\mathbf{B}' = \langle B', C_{B'}\rangle$ *be two belief structure and* $\mathbf{E} = \langle E, C(E)\rangle$ *be the perceived experiences. Then* $\mathbf{B}'$ *is the e-contraction from* $\mathbf{B}$ *with* $\mathbf{E}$ *if and only if*

$$\langle B', C_{B'}\rangle = \langle B \cap E, C_B \setminus C(B \cap E; B \setminus E)\rangle. \tag{3}$$

Proof. It is obvious that (3) is beliefs for a player, i.e., (3) is the event-causality structure of beliefs and that it satisfies (C1).

To prove that (C2) is satisfied, suppose that there is an objective expression $s \in \mathcal{E}^c \cap (\mathrm{Cn}(\mathbf{B}') \setminus (B' \cup C_{B'}))$. It contradicts (3).

Finally we prove that (C3) is satisfied. Now we suppose that another belief structure $\mathbf{B}'' = \langle B'', C(B'')\rangle$ such that $(B \cup C_B) \setminus (B'' \cup C_{B''}) \subsetneq (B \cup C_B) \setminus (B' \cup C_{B'})$ and $\mathcal{E}^c \cap \mathrm{Cn}(\mathbf{B}') = \emptyset$. It implies that there is an objective expression $p \in (B'' \setminus B') \cap B$ with $p \notin \mathcal{E}^c \cap \mathrm{Cn}(\mathbf{B}'')$. Since $B' = B \cap E$, it implies $p \in B \setminus E$ or $p \in E \setminus B$, in contradiction. □

An expansion adds new propositions monotonously. An *e-expansion* inherits this notion in the light of a player's perceived experiences. Consider the same $\mathbf{B}, \mathbf{B}'$, and $\mathbf{E}$ as Definition 4.2, then we define an e-expansion as follows:

Definition 4.4. $\mathbf{B}'$ is *an e-expansion* from $\mathbf{B}$ and with $\mathbf{E}$ if and only if

(E1) $\mathbf{B} \subseteq \mathbf{B}'$;
(E2) $(E \cup C(E)) \subseteq \mathrm{Cn}(\mathbf{B}')$.

While (E1) inherits the notion of the standard expansion, it guarantees that a player does add the experienced propositions at most. Therefore, he does not add any non-experienced propositions such as subjective impressions and misunderstandings. Note that it does not mean that he does not make mistakes because his memory function may lead some mistakes. Here we just say that all the beliefs he holds are derived from the event-causality structures. (E2) guarantees that all his experiences are reflected in the e-expansion.

As the standard contraction and expansion, both e-contraction and e-expansion are supposed that all the player's experiences are consistent with his old beliefs. Nevertheless he may face the facts inconsistent with his held beliefs. In the next, we consider the revision with the perceived experiences, which leads new consistent beliefs even when the experiences are inconsistent with the beliefs initially held by a player.

4.2. **Experience-based Revision.** When a player faces an experience inconsistent with his beliefs, there are usually several alternatives of the revision. For example, suppose that he believes $\{\neg q, p \rightsquigarrow q\}$ and has an experience $\{p\}$. Then he might change his belief to either $\{p, \neg q\}$, $\{p, p \rightsquigarrow q\}$, or $\{p\}$. However, if the player has no reason to change his beliefs, it is reasonable to keep these beliefs. He should change his beliefs only if he has some reason to change them.

Furthermore, if an alternative is no worse than another alternative on all dimensions and better than it on some, then it is more reasonable to prefer the former. That is, the player will follow the Pareto principle. Schulte [18] proposes the revision criterion based on the Pareto principle. If our theory simply applies Schulte's criterion for the revision with the perceived experiences, there are some problems as stated below. Therefore we proposed another revision principle as an *experience-based consistent revision.*

For the definition of the revision principle, we formally state the consistency. We say that a player's belief structure $\mathbf{B}$ is inconsistent if there is an objective expression $p \in \mathbf{B}$ such that $\{p, \neg p\} \subseteq \mathrm{Cn}(\mathbf{B})$ and that $\mathbf{B}$ is consistent if it is not inconsistent. Now consider two belief structures $\mathbf{B} = \langle B, C_B \rangle$ and $\mathbf{B'} = \langle B', C_{B'} \rangle$, and the perceived experiences $\mathbf{E}$ for a player, then we define an experience-based consistent revision as follows:

Definition 4.5 (Revision with Experiences)**.** $\mathbf{B'}$ is *an experience-based consistent revision* (or *an e-revision*) from $\mathbf{B}$ with $\mathbf{E}$ if and only if:

(R1) $(E \cup C(E)) \subseteq \mathrm{Cn}(\mathbf{B'})$;
(R2) $\mathbf{B'}$ is consistent;
(R3) If $s \in B$ and $s \notin B'$, then $\neg s \in \mathrm{Cn}(\mathbf{B'})$;

(R4) There is no other belief state $\mathbf{B}'' = \langle B'', C_{B''} \rangle$ such that $\mathbf{B}''$ satisfies (R1)–(R3) and $(B'' \cup C_{B''}) \vartriangle (B \cup C_B) \subsetneq (B' \cup C_{B'}) \vartriangle (B \cup C_B)$, where $C \vartriangle D := C \setminus D \cup D \setminus C$.

In the definition, (R1) requires that the revised belief structure should reflect his perceived experiences and (R2) guarantees that the belief is consistent. (R4) demonstrates Schulte's revision principle based on the Pareto principle, which requires the minimal change for the e-revision. The notable difference is to employ (R3) following Kraus and Lehmann [14]. (R3) says that, once a player believes a sequence in his environment, he continues to believe it or denies it. By this requirement, the player distinguishes the previously believed sequence from what he did not believe. Namely the difference of his initial belief structure is revealed by the requirement (R3). This is also the central difference with Schulte's Pareto minimal consistent change (See Schulte [18, p. 346]). The example in the next section will make this point clear.

For a set $F \subseteq \boldsymbol{A} \cup \boldsymbol{N}(\boldsymbol{A})$, we define the following two set:

$$\overline{\mathcal{M}}(F) \;=\; \{s \in F \mid \text{the positive expression of } s \text{ is a maximal sequence in } F\}, \text{ and}$$

$$\underline{\mathcal{M}}(F) \;=\; \{s \in F \mid \text{the positive expression of } s \text{ is a minimal sequence in } F\}.$$

Now we characterize the e-revision with the perceived experiences in an info-memory protocol. In the standard revision theory, the expression of the revision is called *Levi Identity* when the revision is expressed by the contraction and expansion. Therefore this following theorem is shown as our version of Levi identity.

Theorem 4.6 (Levi Identity). *Suppose that $\bar{\sigma}$ assigns $\bar{\sigma}(s) = \top$ for all $s \in B$ of $\mathbf{B} = \langle B, C_B \rangle$. $\mathbf{B}' = \langle B', C_{B'} \rangle$ is an experience-based consistent revision from $\mathbf{B} = \langle B, C_B \rangle$ with $\mathbf{E} = \langle E, C(E) \rangle$ if and only if*

$$\mathbf{B}' = \left\langle B \;\cap\; E \;\cup \bigcup_{s \in \overline{\mathcal{M}}(B \setminus E)} \{\neg s\} \cup \underline{\mathcal{M}}(E),\ C_B \setminus C(B \cap E; B \setminus E) \cup C(E) \right\rangle. \quad (4)$$

This theorem states that his beliefs can be revised by the e-contraction for the non-experienced propositions and by the e-expansion of both experiences and negation of non-experienced maximal sequences.

There are two notable assertions in this theorem. The first is that the addition of the negations is not all members but the maximal sequences in $B \setminus E$. As seen later (Proposition 5.1), the consequence set Cn derives the negations of subsequences for each maximal sequence. As it were, the player can reason the consequence of subsequences. Then he does not need to add the negation of non-experienced subsequences for the minimal revision.

The second is related with the first assertion. The addition of the experienced sequences is also the minimal sequences in E for the perceived experiences $\mathbf{E}$. This is also because of the consequences from the possibility connective. Since he believes the possibility $C(E)$, he can derive the supersequences of $s \in E$.

In the next section, we investigate a player's personal view constructed by the revised belief structure after the proof of the theorem.

4.3. **Proof of Theorem 4.6.** First we prove that $\mathbf{B}'$ is an e-revision from $\mathbf{B}$ with $\mathbf{E}$ if $\mathbf{B}'$ is embodied as (4).

For (R1), $s \in \mathrm{Cn}(\mathbf{B}')$ for all $s \in \underline{\mathcal{M}}(E) \subseteq E$ by (4). Since $\mathbf{E}$ is subsequence-closed, $(s \rightsquigarrow t) \in C(E)$ for each super sequence t of $s \in \underline{\mathcal{M}}(E)$. It implies $(s \rightsquigarrow t) \in \mathrm{Cn}(\mathbf{B}')$ for each $s \in \underline{\mathcal{M}}(E)$ and each super sequence $t \in E$ of s. Then, as $\bar{\sigma}(s) = \bar{\sigma}(s \rightsquigarrow t) = \top$, we have $\bar{\sigma}(t) = \top$ for all the super sequence t of s. Hence we obtain $(E \cup C(E)) \subseteq \mathrm{Cn}(\mathbf{B}')$.

For (R2), suppose that $\mathbf{B}'$ is inconsistent. Then there exists an atomic objective expression $s \in \mathbf{B}'$ such that $\{s, \neg s\} \subseteq \mathrm{Cn}(\mathbf{B}')$. Then we have three cases; (i) $s, \neg s \in \mathbf{B}'$, (ii) $s \in \mathbf{B}'$ and $\neg s \in \mathrm{Cn}(\mathbf{B}') \setminus \mathbf{B}'$, or $\neg s \in \mathbf{B}'$ and $s \in \mathrm{Cn}(\mathbf{B}') \setminus \mathbf{B}'$, and (iii) $s, \neg s \in \mathrm{Cn}(\mathbf{B}') \setminus \mathbf{B}'$.

The first case implies that s is a member of a maximal non-experienced sequence as in $\overline{\mathcal{M}}(B \setminus E)$, in contradiction of (4). All the other cases imply that there is a player's belief assignment $\bar{\sigma}$ such that $\bar{\sigma}(s) = \bar{\sigma}(\neg s) = \top$, in contradiction of the definition of the assignment. Then (R2) is satisfied.

For (R3), as $s \in B$ and $s \notin B'$, s is a member of $B \setminus E$. If s is in $\overline{\mathcal{M}}(B \setminus E)$, (R3) is immediately satisfied by (4). Otherwise, for each $r \in (B \setminus E) \setminus \overline{\mathcal{M}}(B \setminus E)$, there is a super sequence $s \in \overline{\mathcal{M}}(B \setminus E)$. As both r and s are also in $\mathbf{B}$ but not in $\mathbf{B}'$, $(r \rightsquigarrow s) \in C_{B'}$). Hence, we obtain $\neg r \in \mathrm{Cn}(\mathbf{B}')$ because $\bar{\sigma}(\neg s) = \bar{\sigma}(r \rightsquigarrow s) = \top$.

For (R4), suppose that there is another revised belief structure $\mathbf{B}''$ satisfying (R1)–(R3) and $(B'' \cup C_{B''}) \vartriangle (B \cup C_B) \subsetneq (B' \cup C_{B'}) \vartriangle (B \cup C_B)$ In the case that $\mathbf{B}''$ is added less than $\mathbf{B}'$, $\mathbf{B}''$ breaks (4). The other case implies that the e-contraction breaks (C3).

For the other direction, we suppose that $\mathbf{B}'$ is an e-revision from $\mathbf{B}$ with $\mathbf{E}$. As the e-contraction guarantees a minimal contraction by (C3), we show that (4) in the e-expansion is a minimal addition to $\mathbf{B}$.

We consider three cases; (i) $\neg s \in B'$ for some $s \in \overline{\mathcal{M}}(B \setminus E)$, (ii) $s \in B'$ for some $s \in \underline{\mathcal{M}}(E)$, and (iii) there is $(s \rightsquigarrow t) \in C(E)$ such that $(s \rightsquigarrow t) \notin C_{B'}$. The first case apparently breaks (R3) and the second case breaks (R1) if there is a minimal sequence $s \in E$ with $s \notin B$. The last case also breaks (R1) because $(s \rightsquigarrow t) \notin \mathrm{Cn}(\mathbf{B}')$

in the case that $\bar{\sigma}(s) = \top$ for a subsequence s of $t \in E$ with $t \notin \underline{\mathcal{M}}(E)$. As a result, if $\mathbf{B}'$ is not the e-revision as (4), then $\mathbf{B}'$ is not the e-expansion.

5. Comparisons with the Kaneko-Kline's Personal View

By the above revision process, a player revises his initial belief structure referring to his perceived experience structure. While the KK theory directly constructs the player's personal view by his memory function, we construct his view from his revised belief structure in the light of the perceived experience. Our construction depends on various the perceived experiences as well as the initial belief structure. Therefore our theory reflects the personality of the players more. Furthermore, we show the conditions of the coincidence with objective information protocols. Through this section, we consider the e-revision of $\mathbf{B}'$ from $\mathbf{B}$ with $\mathbf{E}$ in an objective info-memory protocol $\langle \Pi^o, \pi^o, \mathsf{m}^o \rangle$.

5.1. **Construction of a personal view.** For a start, we study some basic properties of the revised belief structure. By the e-revision, a player obtains some additional information and consequences by his reasoning. He constructs his personal view by making use of the result of his e-revision.

As seen in the previous sections, the e-revision of his belief structure is achieved with minimal change. Indeed the e-revision of the basic belief focuses on the minimal sequences in E_i^C and the maximal sequences in E_i. This is because his reasoning derives the consequence of the relative sequences shown as follows:

Proposition 5.1 (Conceivable Causality)**.** *Let* $\mathbf{B}, \mathbf{B}'$ *be two beliefs and* $\mathbf{E}$ *be experiences for a player. Consider that* $\mathbf{B}'$ *is an e-revision from* $\mathbf{B}$ *with* $\mathbf{E}$*. For two sequences* $s, t \in \mathbf{B}$ *such that* s *is a subsequence of* t*, if* $s \notin \mathbf{B}'$ *and* $\neg t \in \mathbf{B}'$*, then* $\neg s \in \mathrm{Cn}(\mathbf{B}')$*.*

Proof. Since s, $t \in \mathbf{B}$ and s is a subsequence of t and $s \notin \mathbf{B}'$, $s \rightsquigarrow t$ is a member of both $\mathbf{B}$ and $\mathbf{B}'$ by Theorem 4.6. Then, as $\neg t$ and $s \rightsquigarrow t$ is in $\mathrm{Cn}(\mathbf{B}')$, we have $\neg s \in \mathrm{Cn}(\mathbf{B}')$. □

This proposition says that, if a sequence t is a non-experiential belief and the subsequence s of t is also non-experiential, then player i reasons that the subsequence s cannot happen.

The result explicitly shows the central role of a player's reasoning for economy of the size of the belief change. Indeed, if this proposition is not shown, the player is compelled to revise causality of all sequences and the subsequence of them.

In order to derive player i's personal view from the revised belief structure $\mathbf{B}'_i = \langle B'_i, C_{B'_i} \rangle$, we consider the following sets:

- W^i is the set of all information pieces occurring in $\mathrm{Cn}(\mathbf{B}'_i)$;
- A^i is the set of all available actions occurring in a sequence of $\mathrm{Cn}(\mathbf{B}'_i)$.

With the above sets, we define the player i's personal view derived from the revised belief structure $\mathbf{B}'_i$ as follows:

Definition 5.2 (Personal View)**.** $\Pi^i(\mathbf{B}'_i)$ is a *personal information protocol* for player i derived from the belief structure $\mathbf{B}'_i = \langle B'_i, C_{B'_i}\rangle$ such that

$$\Pi^i(\mathbf{B}'_i) = (W^i, A^i, \prec^i, \not\prec^i)$$

where $\prec^i = \{s = \langle \xi, w\rangle \mid s \in \mathrm{Cn}(\mathbf{B}'_i) \cap \boldsymbol{A}\}$ and $\not\prec^i = \{t = \langle \xi, w\rangle \mid \neg t \in \mathrm{Cn}(\mathbf{B}'_i)\}$.

In contrast to the KK theory, our theory has a relation $\not\prec^i$. This represents i's belief that any sequence of $\not\prec^i$ will not occur. While he builds his personal view by $\prec^i$, he has the additional view based on $\not\prec^i$.

In the next example, we see the concrete construction of i's personal view from $\mathbf{B}'_i$. Moreover the example negatively answers the following two remarkable questions:

(i) Can the player derive the same personal view as the objective information protocol?

(ii) Does all the player obtain the same revised belief structure under the same initial belief or under the same experience?

The negative answers shows the natural results that it is difficult to capture an objective world accurately and that each personal view depends on both each personal belief and experiences.

Example 5.3. Let us reconsider the example of the Plato's analogy of the cave in the Introduction. That example is formally represented as the objective information protocol $\Pi^o = (W^o, A^o, \prec^o)$ where $W^o = \{m, H, C, e\}$, $A^o = \{p\}$, and $\prec^o$ is all the subsequences of the maximal sequence $\langle (m,p), (H,p), (C,p), e\rangle$ depicted in Figure 2. Here $p \in A^o$ means that time $p$asses.

For the objective information protocol Π^o, we give the objective player assignment as $\pi^o(w) = \{i\}$ for any $w \in W^o$ and the following memory function of player i:

$$\mathsf{m}_i^o\langle \xi, w\rangle = \begin{cases} \{\langle w\rangle\} & \text{if } w = m, H, C \\ \left\{ \begin{array}{l} \langle (m,s), (H,p), (C,p), e\rangle, \\ \langle (m,r), (C,p), (H,p), e\rangle \end{array} \right\} & \text{if } w = e. \end{cases} \tag{5}$$

In this example, we consider two different initial belief structures $\mathbf{B}^1, \mathbf{B}^2$ dipicted in Figures 8, 9. We give the basic belief B_i^1 in $\mathbf{B}^1$ as $B_i^1 = \{t_1, t_2, t_3, t_4\}$ and B_i^2 in

$\mathbf{B}^2$ as $B_i^2 = \{t_1, t_2\}$ where each maximal sequence is given as follows:

$$\begin{aligned} t_1 &= \langle (m,p),(H,p),(H,p),e \rangle, \\ t_2 &= \langle (m,p),(H,p),(C,p),e \rangle, \\ t_3 &= \langle (m,p),(C,p),(H,p),e \rangle, \\ t_4 &= \langle (m,p),(C,p),(C,p),e \rangle. \end{aligned}$$

Furthermore consider the player's experience $E_i(\Xi_i)$ with $\Xi_i = \{e\}$. Then, for the memory function (5) with $\Xi_i = \{e\}$, his basic experience $E_i(\Xi_i)$ is all the subsequences of the maximal sequences $\langle (m,p),(H,p),(C,p),e \rangle$ and $\langle (m,p),(C,p),(H,p),e \rangle$. We now look at the e-revisions from $\mathbf{B}^1$ and $\mathbf{B}^2$ with $\mathbf{E}$.

When the player's initial belief is $\mathbf{B}^1$, $\mathbf{E}^C$ is the subsequences of t_1 and t_4 except $\langle m \rangle$ and $\langle e \rangle$. Then, Theorem 4.6 shows that he has $\neg t_1$ and $\neg t_4$. As $\mathbf{E} \subseteq \mathbf{B}^1$, he does not have any additional observations.

On the other hand, he has a different revised belief in the case of $\mathbf{B}^2$ even for the same perceived experieces. Indeed, $\mathbf{E}^C$ is the subsequences of only t_1 except $\langle m \rangle$ and $\langle e \rangle$. Hence, he has only $\neg t_1$ and have an additional observation t_3.

This consequence presents that the above questions are negative. Namely, a player does not necessarily derive the same personal view as the objective information protocol. Since the personal view depends on his memory function and the accumulation, he may have some misunderstandings for the objective world. Even if he has the more experiences, he may have an inaccurate personal view in comparison with the objective information protocol.

In addition, even when the player has the same experiences, the revised belief may be different according to his initial belief structure. This is because of (R3) in the e-revision and shows a natural situation that different people can have different views. As the standard belief revision does not require (R3) for the minimal change and regards the new information as important, there is no difference between the initial beliefs in the standard theory (Consider the above cases without (R3)). Our

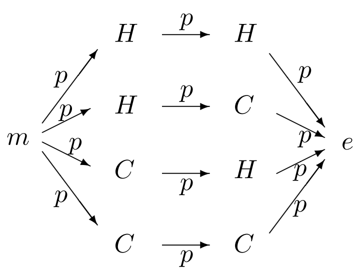

FIGURE 8. Belief $\mathbf{B}^1$

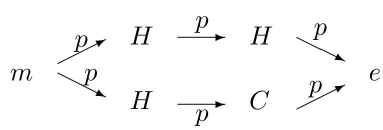

FIGURE 9. Belief $\mathbf{B}^2$

theory can capture the difference of the players' character as the difference of the initial beliefs as well as that of the experiences.

5.2. **Recoverability of the objective info-memory protocols.** As seen in the previous example, the player's personal view does not necessarily coincide with the objective information protocol. Then what conditions need for the coincidence on earth? This subsection investigates the conditions.

For the coincidence with the objective information protocol, we first study the relation between the personal view $\Pi^i(\mathbf{B}')$ derived from $\mathbf{B}'$ and the axioms in the KK theory. This is because the both KK theory and our theory define objective information protocol with the requirements of the axioms. Moreover the basic and non basic axioms guarantee the correspondence to the Kuhn's extensive-form games by the KK theory. Now we introduce the following assumption on i's experience $E_i(\Xi_i)$:

> **Terminal-recognition:** For any $\langle \xi, w \rangle \in E_i(\Xi_i)$ with $w \notin W^{Eo}$ (end pieces in Π^o), there is $\langle \xi', w' \rangle \in E_i(\Xi_i)$ such that $\langle \xi', w' \rangle$ is a supersequence of $\langle \xi, w \rangle$.

This condition states that, for each experience $\langle \xi, w \rangle$ of player i, he has another experience $\langle \xi', w' \rangle$ including $\langle \xi, w \rangle$ if $\langle \xi, w \rangle$ is not a terminal in theobjective information protocol. That is, he can recognize the terminal in the objective situation. It is a substitution in our theory for *action-closedness* in Kaneko and Kline [13, p. 19]. Then the personal information protocol guarantees the basic axioms as follows:

Proposition 5.4 (Preservation of Axioms)**.** *For player i's belief structure* $\mathbf{B}'$*,*

(i) $\Pi^i(\mathbf{B}')$ *satisfies Axioms B1, B2, and B3;*
(ii) $\Pi^i(\mathbf{B}')$ *satisfies Axiom B4 under the terminal-recognition.*

Proof. (i) $\Pi^i(\mathbf{B}')$ satisfies Axiom B1 by the definition.

For Axiom B2, consider $a \in A^i$. Then there is a feasible sequence $(s, a) \prec^i t$ for some $s, t \in W^i$ or $(u, a) \not\prec^i v$ for some $u, v \in W^i$ by the definition.

For Axiom B3, recall that the basic belief B_i of the initial belief structure $\mathbf{B}$ and $E_i(\Xi_i)$ for any $\Xi_i \subseteq \Xi^o$ are subsequence-closed by the definitions. Moreover, for any maximal sequence t in $\in B_i'$ of $\mathbf{B}'$, (a) $t \in \mathrm{Cn}(\mathbf{B}')$ or (b) $\neg t \in \mathrm{Cn}(\mathbf{B}')$. In the case (a), for any subsequence s of t, $s \prec^i t$ because of the subsequence-closedness of B_i and $E_i(\Xi_i)$. In (b), if $\neg t \in \mathrm{Cn}(\mathbf{B}')$, then $t \in \overline{\mathcal{M}}(B \setminus E)$. If a subsequence s is in $E_i(\Xi_i)$, then $s \in \prec^i$. Otherwise $s \in \not\prec^i$ because of Proposition 5.1. Therefore $\Pi^i(\mathbf{B}')$ satisfies Axiom B3.

(ii) Axiom 4 is immediately obtained from the terminal-recognition. □

The above proposition shows that the constructed personal view in our theory is connected with the objective information protocol required in the KK theory. With this result, we investigate the possibility of the coincidence with the objective information protocol by specifying memory functions.

For player i's position $\langle \xi, w \rangle = \langle (w_1, a_1), \ldots, (w_m, a_m), w \rangle \in \Xi_i$, $\langle \xi, w \rangle_i$ is defined as the subsequence of $\langle \xi, w \rangle$ consisting only of i's information piece $w_t \in W^i$. Following Kaneko and Kline [12, pp. 19–20], we provide i's memory functions in $(\Pi^o, \pi^o, \mathsf{m}^o)$ as follows:

- m_i^{PI} is called the m-*perfect-information memory function* if

$$\mathsf{m}_i^{PI}\langle \xi, w \rangle = \{\langle \xi, w \rangle\} \text{ for } \langle \xi, w \rangle \in \Xi_i;$$

- m_i^{PR} is called the m-*perfect-recall memory function* if

$$\mathsf{m}_i^{PR}\langle \xi, w \rangle = \{\langle \xi, w \rangle_i\} \text{ for } \langle \xi, w \rangle \in \Xi_i;$$

- m_i^{C} is called the *classical memory function* if

$$\mathsf{m}_i^{C}\langle \xi, w \rangle = \{\langle \eta, v \rangle | \ \langle \eta, v \rangle \in \Xi \text{ and } v = w\} \text{ for } \langle \xi, w \rangle \in \Xi_i.$$

The m-perfect-information memory function is a memory function that player i remembers all knots that he passes by at each of his positions. Whereas the m-perfect-recall function is a function he remembers all knots where he decided at each of his positions. Therefore he does not remember the knots of his opponents. The classical memory function gives all the histories of his position from the root although the player cannot specify the exact position if several positions have the same tail.

Both the m-perfect-information and classical memory functions will remind the reader of some concepts in the standard game theory such as perfect information games and perfect recall. Indeed we can see the common properties seen in the standard extensive-form games. For instance, the following fact indicates that these two memory functions bring the same situation as the standard one to the players.

Fact 5.5. [Minimal Experiences] Given i's perceived experience structure $\mathbf{E}$ with either class of the above three memory functions, the basic experience $E_i(\Xi_i)$ satisfies the terminal recognition if $\Xi_i = \Xi_i^{Eo}$ where $\Xi_i^{Eo} = \{\langle \xi, w \rangle \in \Xi_i^o \mid w \in W^{Eo}\}$.

This fact shows that the players can recall a sufficient history only by the accumulation of his maximal position. Considering that each of memory functions corresponds to perfect information and perfect recall in the standard one, the assumptions of the extensive-form games imply that the players are given the sufficient information or memories. That is, the players in the games suppose the hypothetical end of the game and they just make decisions of complete contingent plans. Hence we consider the recoverability of an objective situation. Now we define the recoverability as follows:

Definition 5.6. The i's personal information protocol $\Pi^i(\mathbf{B}')$ recovers the objective information protocol $\Pi^o = (W^o, A^o, \prec^o)$ if and only if $\prec^i = \prec^o$.

Then we have a result of the recoverability by the classical memory functions. In addition the recoverability is possible by m-perfect information functions show as the corollary of the classical memory functions as the m-perfect-information memory function is a special case of the classical memory function.:

Proposition 5.7. *Suppose that* $\mathsf{m}_i^o = \mathsf{m}_i^C$. *Then the personal information protocol* $\Pi^i(\mathbf{B}')$ *coincides with the objective information protocol* Π^o *if and only if* $\Xi_i = \Xi_i^{Eo}$

Proof. By Fact 5.5,

$$\bigcup_{\langle \xi, w\rangle \in \Xi_i} \mathsf{m}_i^C \langle \xi, w\rangle = \bigcup_{\langle \xi, w\rangle \in \Xi_i^{Eo}} \mathsf{m}_i^C \langle \xi, w\rangle.$$

Then $E_i(\Xi_i) = E_i(\Xi_i^{Eo}) = \prec^o$. Hence we have $\prec^o \subseteq \mathrm{Cn}(\mathbf{B}')$ by Theorem 4.6. If there is a $p \in \mathrm{Cn}(\mathbf{B}') \setminus \prec^i$, then $p \in \mathrm{Cn}(\mathbf{B}') \cap \boldsymbol{N}(\boldsymbol{A})$. Therefore we have $\prec^i = \prec^o$. □

Corollary 5.8. *Suppose that* $\mathsf{m}_i^o = \mathsf{m}_i^{PI}$. *Then the personal information protocol* $\Pi^i(\mathbf{B}')$ *coincides with the objective information protocol* Π^o *if and only if* $\Xi_i = \Xi_i^{Eo}$.

6. Concluding Remarks

This paper investigates a player's dynamic process to revise and construct his personal view. Our theory presents a general framework to describe the situation that the players look at their environment in different ways. We state the relation with the standard revision theory and give some comments on the further research of our theory below.

As stated in the Introduction, our inductive derivation is a different approach from the literature of the learning models. Moreover, while making use of the framework of the KK theory, our theory focuses on the permanent revision process based on the player's experiences. The treatment of the experiences in out theory is a slight different from the standard belief revision theory.

Belief revision theory has developed in philosophy and computer science in order to inquire how people change their beliefs and knowledge or how databases in computers should be changed. Alchourrón, Gärdenfors and Makinson [1] (so-called

AGM theory) made a major breakthrough in this literature. The AGM theory provides a set of axioms to be satisfied for a belief revision to be reasonable. In the theory, the whole belief that the player holds consists of a basic belief and the consequence of it. The revision in the AGM theory is simultaneously revised without any distinction between the basic belief and the consequence of it.

In contrast to the AGM theory, our theory distinguishes between what a player originally believes and what he logically derives from the original belief such as the basic event set and the causality set or consequence set. This is based on our motivation that the players are not simple a database but have some logical abilities. This approach is close to the belief base theory, which distinguishes between the belief base and the consequence.[5] However the theory of the belief base focuses neither on the new observations as the experiences in our theory nor on the accumulation of them. Our theory combines two parts, the use of accumulated observations and the logical derivations from them, for decision-making.

Recently Board [2] provides a formal dynamic model of interactive reasoning in order to investigate players' beliefs and belief revision in extensive-form games. However, the standard game theory supposes that all the players know their environment sufficiently and look at the same one. Therefore the players reason their actions based on the knowledge of the environment. In contrast to the approach, our theory investigates the formation of players' view of their environment. The players are allowed to have different views on the identical environment.

As this paper aims the provision of a general framework of the revision with the experiences, there remain several studies to be done. In the further research, we investigate (i) how the players with different views make decisions in our theory (ii) where the various players' views are going after the repetitive revision. In the standard game theory, the players face the identical situation and know that even when considering incomplete information games. Our theory is a start to inquire that it is possible to achieve and to analyze misunderstandings pointed by Kaneko [10] and Kaneko and Kline [11]. In the society that people do not necessarily recognize identical environments, how do they harmonize with each other? This is the fount to pursue in our research.

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